Math 10A
Worksheet, Discussion #2; Tuesday, 6/19/2018
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1 Domain

1.1 Example

1. Find the domain of $\frac{x^2+1}{x-5}$.

> **Solution:** For a fraction, the domain of f/g is $\{\text{domain of } f\} \cap \{\text{domain of } g\} \cap \{g \neq 0\}$. So in our case, the domain of $f = x^2 + 1$ and $g = x - 5$ are both all of $\mathbb{R}$ and $\{x | g(x) \neq 0\} = \{x | x \neq 5\}$ so the domain is $(-\infty, 5) \cup (5, \infty)$.

2. Find the domain of $\sqrt{4 - (1 - 2x)^2}$.

> **Solution:** The domain of $\sqrt{4 - x^2}$ is $[-2, 2]$. Then the linear transformation $1 - 2x = -2x + 1$ shifts the graph left by 1 and then reflects it across the y axis and compresses it by a factor of 2. So we go from $[-2, 2]$ to $[-3, 1]$, to $[-1, 3]$ to $[-1/2, 3/2]$.

3. Find the range of $-4\sqrt{x} + 2$.

> **Solution:** The range of $\sqrt{x}$ is $[0, \infty)$ so the range of $-4\sqrt{x}+2$ is $(-4\infty+2, -4\cdot 0+2] = (-\infty, 2]$.

1.2 Problems

4. Find the domain of $\ln(x - 3)$.

> **Solution:** The domain of $\ln x$ is $(0, \infty)$. The linear shift $x - 3$ shifts the function to the right by 3 and so we add 3 to the domain. This means that the domain of $\ln(x - 3)$ is $(0 + 3, \infty + 3) = (3, \infty)$.

5. Find the domain of $\sqrt{3x - 3}$.

Solution: The domain of $\sqrt{x}$ is $[0, \infty)$. The linear shift $3x - 3$ acts by first shifting the graph right by 3, then contracting by a factor of 3. What this does to the domain is add 3 and then divide by 3, so we get $[3, \infty)$ and then $[1, \infty)$ as the final domain.

6. Find the domain of $\sqrt{9 - (2x + 3)^2}$.

Solution: The domain of $\sqrt{9 - x^2}$ is when $9 - x^2 \geq 0$ or when $x^2 \leq 9$ or $x \in [-3, 3]$. Then applying the linear shift $2x + 3$ first shifts the domain to the left by 3 and then compresses it by a factor of 2. Doing so gives $[-3, 3]$, then $[-6, 0]$, then finally $[-3, 0]$.

7. Find the range of $2e^x + 1$.

Solution: The range of e^x is $(0, \infty)$. The linear shift first stretches the range by 2 then shifts it up 1 to get $(0, \infty)$ again and then $(1, \infty)$ as the final range.

8. Find the range of $6 \sin x + 3$.

Solution: The range of $\sin x$ is $[-1, 1]$. The range after multiplying it by 6 and adding 3 is $[-6, 6]$ and then $[-3, 9]$.

9. Find the range of $-2 \cos x + 1$.

Solution: The range of $\cos x$ is $[-1, 1]$ and multiplying by -2 gives $[-2, 2]$. Finally we add 1 to the range to get $[-1, 3]$.

1.3 Extra Problems

10. Find the domain of $\frac{1}{2x+1}$.

Solution: The domain is when the denominator is nonzero, so when $2x + 1 \neq 0$ or $x \neq \frac{-1}{2}$. So the domain is $\mathbb{R} \backslash \{-1/2\}$.

11. Find the domain of $\sqrt{3 - x}$.

Solution: The domain is when $3 - x \geq 0$ or when $x \leq 3$, meaning $(-\infty, 3]$.

12. Find the domain of e^{2x+1}.

Solution: The domain of e^x is everything and so shifting it to the left and right and compressing it doesn't change it. So the domain is $\mathbb{R}$.

13. Find the range of $5\arctan(x) - \pi$.

Solution: The range for $\arctan(x)$ is $(-\pi/2, \pi/2)$ so the range of $5\arctan(x) - \pi$ is $(-5\pi/2 - \pi, 5\pi/2 - \pi) = (-7\pi/2, 3\pi/2)$.

14. Find the range of $\frac{3}{x+2} - 1$.

Solution: The range of $\frac{1}{x}$ is $\mathbb{R}\backslash\{0\}$. The shift of $x+2$ changes the domain but not the range. Then multiply by 3 and subtracting 1 gives a range of $\mathbb{R}\backslash\{3\cdot 0-1\} = \mathbb{R}\backslash\{-1\}$.

2 Inverses

2.1 Example

15. Find the inverse and the domain and range of e^{x^2} on $(-\infty, 0]$.

Solution: We are told the domain is $(-\infty, 0]$. So $x \leq 0$ and $x^2 \geq 0$ so $e^{x^2} \geq e^0 = 1$ so the range is $[1, \infty)$. Another way to do this is to find the domain of the inverse. To find the inverse, we set $x = e^{y^2}$ and solve for y to get $y^2 = \ln(x)$ and $y = -\sqrt{\ln(x)}$. The reason we use the negative square root is because the domain of the function is negative, so the range of the inverse should also be negative. Now for the domain of the inverse, we need $\ln(x) \geq 0$ so $x \geq 1$ and the domain of the inverse is $[1, \infty)$.

2.2 Problems

16. Find the inverse and the domain and range of $y = (x + 1)^3 - 2$.

Solution: The domain is all real numbers. Inverting it gives $x = (y+1)^3 - 2$ or $x + 2 = (y+1)^3$ and $y = \sqrt[3]{x+2} - 1$ as the inverse. The domain of this is also all real numbers so the range of the original function is all real numbers.

17. Find the inverse and the domain and range of $y = \frac{x}{1-x}$.

Solution: The domain is all numbers except $x = 1$ because we cannot divide by 0 so the domain is $\mathbb{R}\backslash\{1\}$. The inverse is gotten by solving $x = \frac{y}{1-y}$ so $x - xy = y$ and $x = y + xy = y(1+x)$ so $y = \frac{x}{1+x}$. The domain of this is $\mathbb{R}\backslash\{-1\}$ because we cannot divide by 0 and this is also the range of the original function.